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Hadronic decay of a scalar *B* meson from the lattice

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We explore the transitions $B(0^+)$ to $B\pi$ and $B_s(0^+)$ to BK from lattice QCD with $N_f = 2$ flavors of sea quark, using the static approximation for the heavy quark. We evaluate the effective coupling constants, predicting a $B(0^+)$ to $B\pi$ width of around 160 MeV. Our result for the coupling strength adds to the evidence that the $B_s(0^+)$ meson is not predominantly a molecular state (BK).

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I. INTRODUCTION

The interest in the excited states of heavy-light mesons has been enhanced by the striking discovery that the $c\bar{s}$ states with $J^P = 0^+$ and 1^+ have very narrow widths [1–4]. This raises the question of whether the corresponding $b\bar{s}$ states will also be narrow. The main reason for the narrow width of D_s mesons is that the transition to DK is not energetically allowed (for the 2317 MeV state) or the state is close to threshold (for the 2457 MeV state). Thus the only allowed hadronic decay proceeds via isospin violation (since $m_d \neq m_u$) to $D_s\pi$ and will have a very small width. Likewise, if the equivalent $b\bar{s}$ states are close to or below the BK threshold, then they will be very narrow.

Lattice studies have addressed the energies of these P-wave $b\bar{s}$ states [5–7] and concluded that they indeed lie close to or below threshold and hence have very small decay widths. Although the lattice studies use $b\bar{s}$ creation operators for these states, it is also possible that a molecular description (as a BK bound state) is more appropriate, as has been suggested for the $c\bar{s}$ case [8]. To clarify this situation further, it would be very useful to evaluate the hadronic transition strength from the scalar B state to a B meson plus a light pseudoscalar meson.

Here we evaluate these hadronic transition amplitudes using lattice methods. This has relevance to the decay of a scalar B or B_s meson to B plus a pseudoscalar meson.

II. SPECTRUM

In the heavy quark limit, the $\bar{Q}q$ meson, which we refer to as a "B" meson, will be the "hydrogen atom" of QCD. Since the meson is made from nonidentical quarks, charge conjugation is not a good quantum number. States can be labeled by $L_\pm$, where the coupling of the light quark spin to the orbital angular momentum gives $j_q = L \pm \frac{1}{2}$. In the heavy quark limit these states will be doubly degenerate since the heavy quark spin interaction can be neglected, so the S state will have $J^{PC} = 0^-, 1^-$ while the $P-$ state will have $J^P = 0^+, 1^+$. Note that in the static case, the self energy of the static heavy quark is unphysical, so that only mass differences are physical. Our notation for these static-light mesons is $B(nL_\pm)$ where $n = 1$ (often omitted) is the ground state, and $n = 2$ the first excited states, etc. Here we will be studying the transition from the P_- state to the S state emitting a pion in a relative S-wave. This can be applied to the decays $B(0^+) \to B(0^-)\pi$ and to $B(1^+) \to B(1^-)\pi$.

We shall be using the $N_f = 2$ lattice configurations [9] with $\beta = 5.2$ and volume $16^3 \times 32$ with SW-clover improvement coefficient 2.0171. We only use the unitary points, namely, those with valence light quarks of the same mass as the sea quarks. The details of the spectrum from Ref. [7] are collected in Table I.

The method we shall use to obtain three-point correlations (next section) using time slice random sources can be used for two-point correlations and compared with the maximal variance reduction (MVR) method [5] used for the two-point correlators in extracting the spectrum [7]. For our lighter quark mass, we find the local-local $B(S)$ correlator is more precisely determined for $t > 4$ by 40 gauge configurations of MVR than 100 gauge configurations of time slice evaluation, although the latter had a somewhat smaller computational overhead. Since larger t is important for separating ground states and excited

TABLE I. Lattice parameters and results from Ref. [7] for the energies of $Q\bar{q}$ states in units of r_0 for dynamical fermions with $N_f = 2$. The values of r_0/a and the $q\bar{q}$ pseudoscalar meson mass are from Ref. [9]. Here we set the scale using r_0 of 0.525(25) fm. The heavy-light meson lattice energies contain the static source self-energy so that only differences are physical.

κ	0.1355	0.1350
MVR gauges	40	20
t slice gauges	100	20
r_0/a	5.041(40)	4.754(40)
$r_0 m(0^{-+})$	1.48(3)	1.93(3)
$r_0 m(1S)$	3.73(8)	3.68(7)
$r_0 m(2S)$	5.60(14)	5.61(8)
$r_0 m(1P_-)$	4.75(6)	4.71(8)
$r_0 m(2P_-)$	7.38(9)	7.1(2)

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states, MVR is the method of choice for the two-point correlation. Because it does not generalize efficiently to the three-point correlation, we use the time slice method there.

III. DECAY TRANSITIONS

Following the methods [10,11] used to study the hadronic transitions such as hybrid decay and ρ to $\pi\pi$, we can determine the transition amplitude provided that there is approximate equality of energies between the initial state $B(1P_-)$ and the final two-body state $B(1S)$ π. Here we are taking the b quark as static and using two flavors of light quark. Staying within the fully unitary sector of the theory, we can study transitions with the same valence quarks in the B mesons and pion as in the sea.

The lightest two-body $B(1S)$ π state on a lattice will be when the pion has relative momentum zero. The energy differences are then given by $a\Delta E = 0.09, 0.19$ for light quarks of mass corresponding approximately to $2m_s/3$ and m_s for $\kappa = 0.1355$ and 0.1350 respectively [7]. Especially for the lighter quark case, this is an energy difference small enough to apply the method [10,11], namely $\Delta Et \ll 5$, up to large t-values.

We need to evaluate correlations between $B(1P_-)$ at $t = 0$ and $B(1S)$ π at time t. This involves quark propagators between three space-time points. This is illustrated in Fig. 1. The heavy quark propagator, however, is trivial to evaluate: as a product of gauge links with a $(1 \pm \gamma_4)/2$ projector for spin. We create $B(S)$ as $Q\gamma_5\bar{q}$ and $B(P_-)$ as $Q\bar{q}$, and, in both cases, also two different fuzzed versions of these [5,7]. In this exploratory study, we only consider a pion with zero momentum (so we sum over relative spatial position) with a local creation operator $q\gamma_5\bar{q}$.

To gain sufficient statistics for the three-point correlations, we wish to evaluate the correlation using every space and time point on the lattice as a source. To achieve this, we follow the stochastic technique used previously [7,12]. We use a stochastic source ξ (complex Gaussian random number in every color, Dirac, space component) at a given time slice t. We then evaluate the propagator ϕ from this source using $M\phi = \xi$ where M is the Wilson-Dirac matrix for the light quark. The required correlation can then be obtained from this propagator, schematically as,

$$C_3(t' - t) = \phi^*(t', y)_{ai}\phi(t, y)_{bj}(1 + \gamma_4)_{ji} \times [\Pi_{t''=t,t'-1}U(y, t'')]_{ab} \quad (1)$$

where all repeated indices (and y) are summed. The product of stochastic sources implicit in the product of ϕ's then gives an expectation value which is the required sum over pion sources throughout the time slice at t, whereas the noise terms average to zero. By using more independent stochastic samples, the average over them (from Eq. (1)) will have reduced noise. One can also improve the signal to noise ratio by combining results from different time values t for the stochastic source.

In practice we used one stochastic sample per time slice, but all time slices in turn, as in Ref. [11]. This implies 32 inversions per gauge to evaluate the required three-point correlation from all sources to all sinks. This is computationally very efficient and provides sufficient precision, as shown below. We use 100 gauges for the lighter quark mass and 20 for the heavier.

The motivation for using a source restricted to one time slice is to ensure that the noise contributions decrease as the signal decreases with increasing $|t' - t|$. Note that a two-point correlator, for example, for the pion with zero momentum and local creation and destruction operator, can be obtained likewise from

$$C_\pi(t' - t) = \phi^*(t', y)_{ai}\phi(t', y)_{ai}. \quad (2)$$

In this work, we extract the ground state pion contribution to C_π from a fit to pion correlations obtained from conventional analyses [9] with nonstochastic sources, so we do not use the stochastic result for the two-point pion correlators, other than as a check.

Likewise, for $B(P_-)$ with local creation and destruction operators, the two-point correlator can be obtained from

$$C_{B(P_-)}(t' - t) = \phi^*(t', y)_{ai}\xi(t, y)_{bj}(1 + \gamma_4)_{ji} \times [\Pi_{t''=t,t'-1}U(y, t'')]_{ab}. \quad (3)$$

As we discussed above, this latter expression for C_B is more noisy than the MVR method at larger $|t' - t|$, so we again only use it as a cross check.

The normalized transition amplitude x on a lattice can then be obtained from the ratio

$$\frac{C_3(t)}{\sqrt{C_{B(P_-)}(t)C_\pi(t)C_{B(S)}(t)}} = xt + \text{const} \quad (4)$$

provided that the transition rate is not too large, namely $xt \ll 1$. This ratio for the decay to π^+ is plotted for our lighter quark mass in Fig. 2. As well as illustrating the result for each of our three operators to create a heavy-light meson, we can choose to improve the ground state projection of the $B(S)$ and $B(P_-)$ by using an appropriate

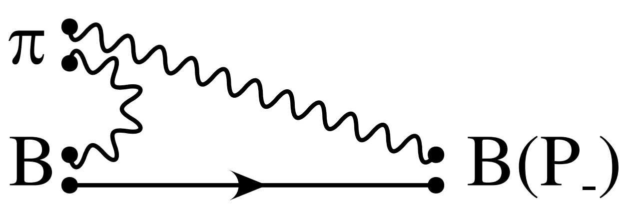

FIG. 1. Quark diagram evaluated for the transition from $B(P_-)$ to $B\pi$. Here $B(P_-)$ is the static-light meson that comprises degenerate scalar and axial B mesons.

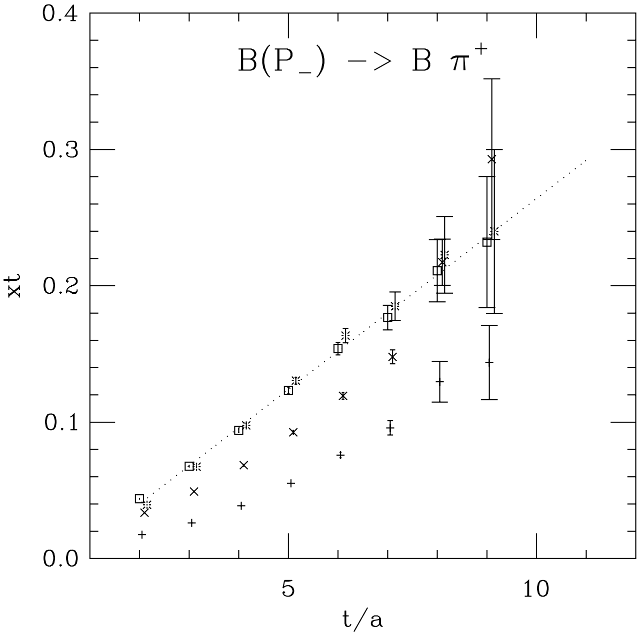

FIG. 2. Normalized three particle correlator versus t/a for $\kappa = 0.1355$. The points marked ($+$, $\times$, $*$) are for local, lightly fuzzed and heavily fuzzed operators, respectively. The combination which optimizes the ground state is shown by squares, and a linear fit to it is shown.

linear combination of local and fuzzed operators. The ratio for this improved projection is also illustrated.

This result shows a linear behavior, as expected if excited state contributions are not significant. We can then read off the hadronic transition amplitude ax from the slope—obtaining $ax = 0.028(3)$.

This is the transition with lattice normalization and for one quark diagram. For the transition $B(P_-) \to B(S)\pi$, there will be two quark diagrams contributing (since either a u or d quark pair can be produced, yielding π^+ or π^0) and the overall rate will be $3/2$ that evaluated from the amplitude x above. To derive the appropriate normalization [10], consider the decay width, even though the decay is not energetically allowed with our parameters. Then $\Gamma = 2\pi x^2 \rho$ where ρ is the two-body phase space in a finite volume which evaluates to $\rho = L^3 k E_\pi/(2\pi^2)$. We have an isotropic decay (S-wave) and it is reasonable to assume that x is independent of the decay momentum k.

To increase predictive power, we evaluate the coupling constant in an effective Lagrangian ($\lambda \bar{B}(P_-)B(S)\pi$) for the three-point vertex describing the decay. The coupling constant λ has the dimensions of mass and in the heavy quark limit we need $\lambda \sim M_B$. Thus the effective coupling is λ/M_B. Then, in infinite volume, this effective coupling squared is proportional to Γ/k, and we use that definition as an effective coupling strength since it is independent of normalization conventions. We then obtain

$$\Gamma/k = 3(L/a)^3(ax)^2 aE_\pi/(2\pi) = 0.46(9) \qquad (5)$$

where for the pion at zero momentum, we may use its lattice mass [9,13] for E_π.

In order to explore the dependence on the light quark mass, we use another quark mass, although we are limited by the need to keep the transition approximately on mass shell, so with decay products of similar energy to the initial scalar meson. We used $\kappa = 0.1350$ where the quark mass is approximately strange. We find a similar plot (Fig. 3) of xt versus t/a with a slope of 0.0237. Since in this case we have a somewhat bigger mismatch (namely $a\Delta E = 0.19$) between the energies for the two-body state and of the scalar meson, we can correct for this by using a two state model [11]. This shows that we would expect some small curvature, even in the ideal case when there is no contribution from excited states. Our lattice result is quite consistent with this curvature. Because of the additional analysis needed to cope with the larger energy gap, the systematic errors from possible excited state contributions are less under control in this case. To allow for this we increase the error estimate, obtaining $ax = 0.024(4)$. Then the effective coupling strength is $\Gamma/k = 0.46(9)$, exactly the same value as obtained at the lighter quark mass.

Since a quark-antiquark pair is created in the decay, it might be expected that the amplitude to produce heavier quarks was smaller. However, a major component of the transition amplitude may come from considerations of the overlap of the initial and final states, and this does not

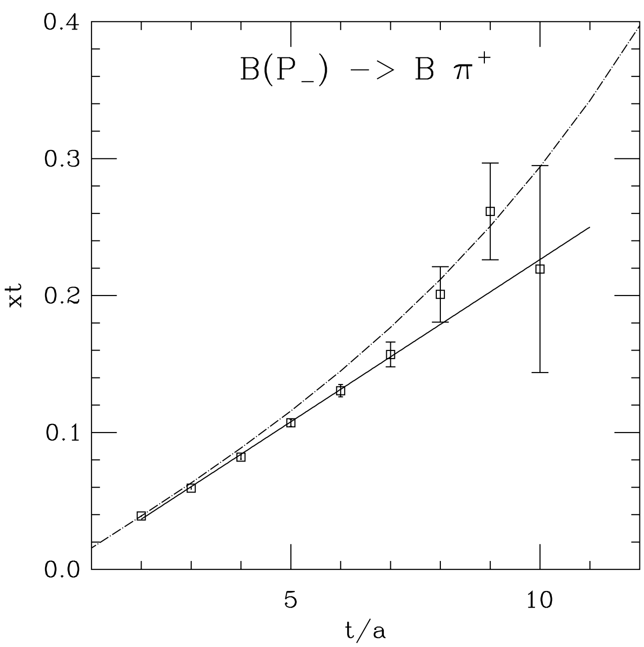

FIG. 3. Normalized three particle correlator versus t/a for $\kappa = 0.1350$. The combination which optimizes the ground state is shown together with a linear fit. We also show the result of a two state model with the correct energy difference ($a\Delta E = 0.19$) between the two-body and scalar B states.

depend on the light quark mass in any very simple way. Indeed in our study [11] of ρ decay to two pions, we saw some evidence that the decay amplitude was largely independent of the light quark mass. This is what we find here for scalar decays also.

The method we have used to evaluate the hadronic transition is only approximate, and assumes that the transition amplitude x is reasonably small and that only transitions to one two-body state are important. Since $xa \approx 0.03$, the magnitude xt is indeed sufficiently small for the range of t-values used, as required. The next energy level (with π of momentum $2\pi/L$) lies higher by $r_0 \Delta E = 1.0$ which is thus approximately three times further in energy from the energy of the decaying meson that the two-body level we include. In general, however, one can proceed in a rigorous way. This involves determining the energy of the two-body system $(B(1S) + \pi)$ as accurately as possible with a full QCD lattice simulation, and then obtaining the dependence of this on the lattice spatial volume. This then gives information on the scattering phase shift in the two-body channel [14–17]. It is, of course, consistent to treat the static quark as quenched, but all the light quarks need to be treated dynamically. In our approach with $N_f = 2$ flavours of degenerate sea quark, this would allow a study of the transition from $B(1P_-)$ to $B(1S) + \pi$. To have the most accurate determination of the two-body energy, one should use a variational approach with both two-body and one body operators. This will involve the three body correlation we have measured above, but also the two to two and box quark diagrams. Our preliminary study indicates, as was found for the case of ρ decay [11], that these diagrams are too noisy to yield sufficiently accurate results, even measuring from all space-time sources for 100 gauge configurations.

IV. DISCUSSION

As discussed above, we are able to measure the transition amplitude from the 0^+ $b\bar{q}$ meson to $B\pi$, provided that the light quark masses are such that the initial and final states have very similar energies. For the case we have explored, with $N_f = 2$ flavors of degenerate light quark, this implies that we must extrapolate in the light quark mass to make contact with the experiment. This we do by assuming that the coupling constant for the transition, as described by an effective Lagrangian, is independent of the light meson mass. This leads to the assumption that the effective coupling strength Γ/k introduced above will be independent of the light quark mass. We do indeed see some evidence from our lattice results that this is the case. Thus we shall use our lattice results for the reduced width, evaluated where no decays are allowed, to compare with experiment and to make predictions. Since we work at a fixed lattice spacing, we are unable to estimate the systematic error arising from not taking the continuum limit.

There is a state known experimentally [18] which is a candidate for the 0^+ $b\bar{q}$ meson, namely, the B^{**} with mass 5698(8) MeV and width 128(18) MeV. This corresponds to an effective coupling strength of $\Gamma/k = 0.34(5)$. However, the experimental state may be a superposition of several states, so mass values and widths for the 0^+ state are not really known experimentally.

From lattice studies with static quarks, the excitation energy of the scalar B^{**} state is estimated [7] to be 368 ± 31 MeV, where this energy difference was evaluated for strange light quarks, but was expected to be similar for nonstrange light quarks. Using this central value of 368 MeV for the energy release, the width of the scalar B^{**} state, with decay to $B\pi$, would be 162(30) MeV. Our result is significantly lower than that obtained [19,20] using a chiral symmetry between the $0^\pm$ B mesons, namely, a width of around 500 MeV using $G_A \approx 1$.

It may also be relevant to compare with experimental data on decays of heavy-light mesons with charm quarks, since there is a wider range of data available [1–4,18]. From the observed [4] mass of $2308 \pm 17 \pm 15 \pm 28$ MeV for $D(0^+)$ and width of $276 \pm 21 \pm 18 \pm 60$ MeV for decay to $D(0^-) + \pi$, one gets $\Gamma/k = 0.73^{+28}_{-24}$. This is a somewhat larger effective coupling strength than the value of 0.46(9) that we obtained above (but consistent within errors), although our evaluation is for static quarks whereas charm quarks are known to be sufficiently light that this can be a poor approximation for them.

It is also possible to extract an effective coupling strength for the decay of $K(1412)$ to $K\pi$, obtaining [18] $\Gamma/k = 0.48(5)$. Thus the experimental data are consistent with an effective coupling strength of about 0.5 for decays of scalar heavy-light mesons with heavy quarks that are b, c, and s. This is very consistent with our *ab initio* evaluation which gives around 0.5 also.

For the $b\bar{s}$ excited mesons, in the limit of degenerate u and d quarks, there will be no decay to pions and the main hadronic decay will be to BK with the creation of a light quark-antiquark pair. In this case our evaluation is partially quenched, in the sense that the strange quark in the B_s meson and K meson is not present in the sea. For the decay of a scalar B_s meson, the energy release may be small or the state may even be stable [7]. Even if the state is stable under strong interactions, we can still evaluate the hadronic transition strength as an effective coupling. Consider the transition $B_s(P_-) \to B(S)K$, there are again two quark diagrams, now with equal weight. Our result is then that $\Gamma/k = 0.61(12)$. If this scalar meson does lie above threshold, we predict a width given by that expression.

Consider now whether the $B_s(P_-)$ meson is a quark-antiquark state or a BK meson. Since we have found a nonzero transition amplitude (our x) on a lattice it follows that the meson and the two-body state mix. Indeed when the meson mass is degenerate with the two-body energy, there will be an avoided level-crossing, with full mixing. What is more significant, however, is the situation in a large volume, when the two-body energy spectrum becomes continuous.

The situation in lattice studies is then more like in experiment—one has to deduce the composition of a hadron from its observed properties. There are lots of extra clues available in lattice studies, however: (i) the mass of the state can be explored as the quark mass varies, (ii) the wave-function and charge form factor of the state can be measured, (iii) the coupling strength of transitions can be evaluated. For the $B(P_-)$ meson, lattice studies with $N_f = 2$ show a spectrum [7] with a mass which is more or less independent of the two-body ($B\pi$) threshold which would not be expected for a molecular state. For a predominantly molecular state (e.g. made of BK or $B\pi$), the decay dynamics to the two-body channel involves a spatial rearrangement, but no quark pair creation. We find a coupling strength, as discussed above, which is similar to that for the scalar decay of $K(1412)$ to $K\pi$, where a molecular structure is not expected, so this implies that a molecular structure is not necessary to explain the scalar B meson decay strength we find. The charge form factor has only been measured [21] for $N_f = 2$ for the ground state $B(S)$ although quenched results [5] show a Bethe Salpeter wave function for $B(P_-)$ similar to quark model expectations. A more definitive lattice conclusion must await studies with lighter quarks, but all the evidence at present points to the heavy-light scalar meson as not being predominantly a molecular state.

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